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# Distribution of Partons in the Transverse Plane

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# (brief) Motivation

DIS  $\xrightarrow{\text{opt.theorem}}$  forward Compton amplitude  $\xrightarrow{Bj-limit} q(x_{Bj})$

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle p | \bar{q} \left( -\frac{x^-}{2}, \mathbf{0}_\perp \right) \gamma^+ q \left( \frac{x^-}{2}, \mathbf{0}_\perp \right) | p \rangle e^{ix^- x_{Bj} P^+}$$

- Light-cone coordinates  $x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^1)$
- $q(x)$  = light-cone momentum distribution of quarks in the target;  $x$  = (light-cone) momentum fraction
- no information about position of partons!

# (brief) Motivation

- generalization to  $p' \neq p \Rightarrow$  **Generalized Parton Distributions**

$$GPD(x, \xi, t) \equiv \int \frac{dx^-}{2\pi} \langle p' | \bar{q} \left( -\frac{x^-}{2}, \mathbf{0}_\perp \right) \gamma^+ q \left( \frac{x^-}{2}, \mathbf{0}_\perp \right) | p \rangle e^{ix^- x P^+}$$

with  $\Delta = p - p'$ ,  $t = \Delta^2$ , and  $\xi(p^+ + p^{+'}) = -2\Delta^+$ .

- can be probed e.g. in **Deeply Virtual Compton Scattering** (DVCS) at HERMES, JLab@12GeV, COMPASS, eRHIC, ...
- Interesting observation: X.Ji, PRL78,610(1997)

$$\langle J_q \rangle = \frac{1}{2} \int_0^1 dx x [H_q(x, 0, 0) + E_q(x, 0, 0)]$$

$$\boxed{\text{DVCS}} \Leftrightarrow \boxed{\text{GPDs}} \Leftrightarrow \boxed{\vec{J}_q}$$

- But: what other “physical information” about the nucleon can we obtain by measuring/calculating GPDs?

# Outline

- Brief overview on generalized parton distributions (GPDs)
- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\Delta_{\perp}^2) \longrightarrow q(x, \mathbf{b}_{\perp})$
  - $\tilde{H}(x, 0, -\Delta_{\perp}^2) \longrightarrow \Delta q(x, \mathbf{b}_{\perp})$
  - $E(x, 0, -\Delta_{\perp}^2) \longrightarrow \perp$  distortion of PDFs when the target is transversely polarized
- ↪ Sivers effect for single spin asymmetries
  - The physics of  $2\tilde{H}_T + E_T$ 
    - correlation between quark angular momentum and quark spin
    - intuitive origin of Boer-Mulders function  $g_1^{\perp}(x, \mathbf{k}_{\perp})$
- Summary

# Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of  $t$ , w.r.t. the average momentum fraction  $x = \frac{1}{2} (x_i + x_f)$  of the active quark

$$\begin{aligned} \int dx H_q(x, \xi, t) &= F_1^q(t) & \int dx \tilde{H}_q(x, \xi, t) &= G_A^q(t) \\ \int dx E_q(x, \xi, t) &= F_2^q(t) & \int dx \tilde{E}_q(x, \xi, t) &= G_P^q(t), \end{aligned}$$

- $x_i$  and  $x_f$  are the momentum fractions of the quark before and after the momentum transfer

- $2\xi = x_f - x_i$

- formal definition:

$$\begin{aligned} \int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right\rangle &= H(x, \xi, \Delta^2) \bar{u}(p') \gamma^+ u(p) \\ &+ E(x, \xi, \Delta^2) \bar{u}(p') \frac{i\sigma^{+\nu} \Delta_\nu}{2M} u(p) \end{aligned}$$

# Generalized Parton Distributions (GPDs)

- measurement of the quark momentum fraction  $x$  singles out one space direction (the direction of the momentum)
  - ↪ makes a difference whether the momentum transfer is parallel, or  $\perp$  to this momentum
  - ↪ GPDs must depend on an additional variable which characterizes the direction of the momentum transfer relative to the momentum of the active quark  $\longrightarrow \xi$ .
- in the limit of vanishing  $t$  and  $\xi$ , the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x, 0, 0) = q(x) \qquad \tilde{H}_q(x, 0, 0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction**  $x$ 
  - ↪  $t$  dependence of GPDs for fixed  $x$ , provides information on the **position space distribution** of quarks carrying a certain momentum fraction  $x$

# Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+q$	$Q$	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x, \xi, t)$	?

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$\int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x, 0, t)$	$q(x, \mathbf{b}_\perp)$

$q(x, \mathbf{b}_\perp)$  = impact parameter dependent PDF



# Impact parameter dependent PDFs

- define state that is localized in  $\perp$  position:

$$|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |p^+, \mathbf{p}_\perp, \lambda\rangle$$

Note:  $\perp$  boosts in IMF form Galilean subgroup  $\Rightarrow$  this state has

$$\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$$

(cf.: working in CM frame in nonrel. physics)

- define **impact parameter dependent PDF**

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+ x^-}$$

# GPDs $\longleftrightarrow q(x, \mathbf{b}_\perp)$

↪ nucleon-helicity nonflip GPDs can be related to distribution of partons in  $\perp$  plane

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i \Delta_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\Delta_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i \Delta_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\Delta_\perp^2), \end{aligned}$$

- no rel. corrections to this result! (Galilean subgroup of  $\perp$  boosts)
- $q(x, \mathbf{b}_\perp)$  has probabilistic interpretation, e.g.

$$\begin{aligned} q(x, \mathbf{b}_\perp) &\geq |\Delta q(x, \mathbf{b}_\perp)| \geq 0 \quad \text{for } x > 0 \\ q(x, \mathbf{b}_\perp) &\leq |\Delta q(x, \mathbf{b}_\perp)| \leq 0 \quad \text{for } x < 0 \end{aligned}$$

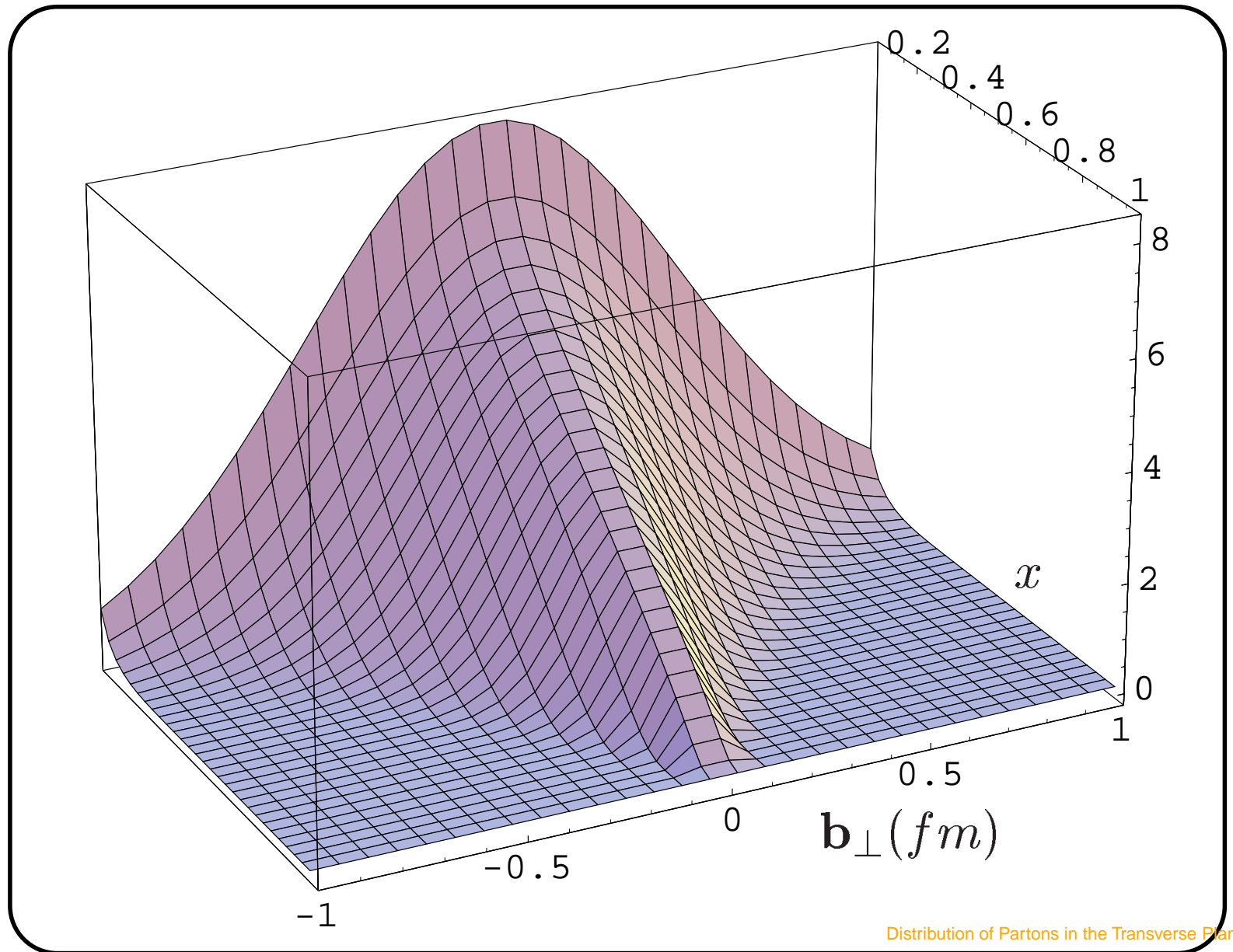
- Note that  $x$  already measures longitudinal momentum of quarks

↪ no simultaneous measurement of long. position of quarks

# GPDs $\longleftrightarrow q(x, \mathbf{b}_\perp)$

- $\mathbf{b}_\perp$  distribution measured w.r.t.  $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$ 
  - $\hookrightarrow$  width of the  $\mathbf{b}_\perp$  distribution should go to zero as  $x \rightarrow 1$ , since the active quark becomes the  $\perp$  center of momentum in that limit!
  - $\hookrightarrow H(x, 0, -\Delta_\perp^2)$  must become  $\Delta_\perp^2$ -indep. as  $x \rightarrow 1$ . Confirmed by recent lattice studies (QCDSF, LHPC)
- Anticipated shape of  $q(x, \mathbf{b}_\perp)$ :
  - large  $x$ : quarks from **localized** valence ‘core’,
  - small  $x$ : contributions from **larger** ‘meson cloud’
  - $\hookrightarrow$  expect a gradual increase of the  $t$ -dependence ( $\perp$  size) of  $H(x, 0, t)$  as  $x$  decreases
- very simple model:  $H_q(x, 0, -\Delta_\perp^2) = q(x) e^{-a \Delta_\perp^2 (1-x) \ln \frac{1}{x}}$ .

# $q(x, \mathbf{b}_\perp)$ in a simple model



# Transversely Distorted Distributions and $E(x, 0, -\Delta_{\perp}^2)$

- So far: only unpolarized (or long. pol.) nucleon! In general ( $\xi = 0$ ):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta_{\perp}^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, 0, -\Delta_{\perp}^2).$$

- Consider nucleon polarized in  $x$  direction (in IMF)

$$|X\rangle \equiv |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \uparrow\rangle + |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \downarrow\rangle.$$

- ↪ unpolarized quark distribution for this state:

$$q(x, \mathbf{b}_{\perp}) = \mathcal{H}(x, \mathbf{b}_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} E(x, 0, -\Delta_{\perp}^2) e^{-i\mathbf{b}_{\perp} \cdot \Delta_{\perp}}$$

- Physics:  $j^+ = j^0 + j^3$ , and left-right asymmetry from  $j^3$  !

# Intuitive connection with $\vec{L}_q$

- (some) DIS-kinematics (target rest frame & momentum transfer in  $-\hat{z}$  direction):

$$\mathbf{p}_{e\perp} = \mathbf{p}'_{e\perp} = \mathbf{0}_\perp \quad p_e^z \rightarrow -\infty \quad p_e^{z'} \rightarrow -\infty$$

- ↪ only “−” component of all momenta and momentum transfers large, e.g.

$$p_e^- \equiv \frac{1}{\sqrt{2}} (p_e^0 - p_e^z) \rightarrow \infty \quad p_e^+ \equiv \frac{1}{\sqrt{2}} (p_e^0 + p_e^z) = \frac{m_e^2 + \mathbf{p}_{e\perp}^2}{2p_e^-} \rightarrow 0$$

- ↪ only “−” component of the electron current  $j_e^\mu = \bar{u}(p')\gamma^\mu u(p)$  large

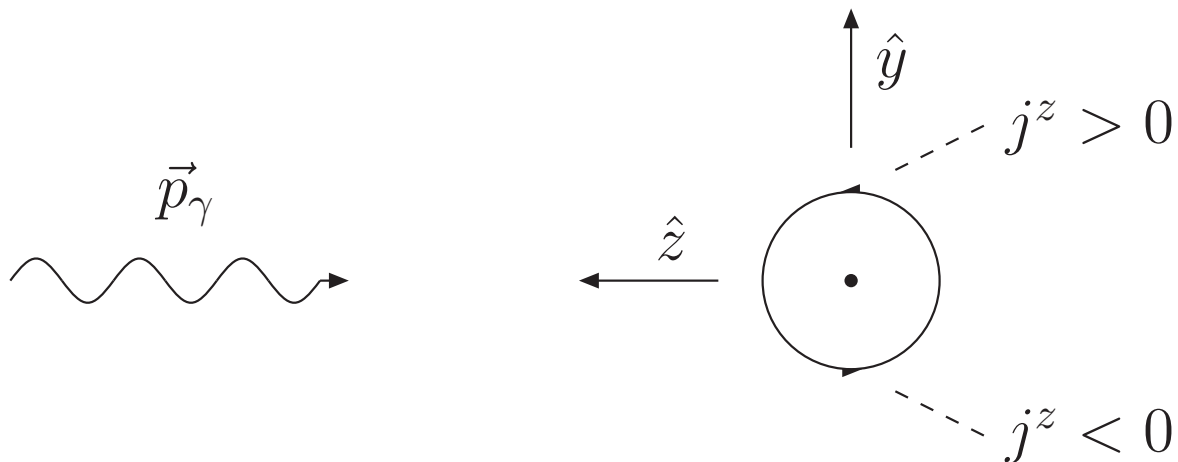
- vector-vector interaction

$$\mathcal{M} \propto j_e^\mu j_q^\nu g_{\mu\nu} = j_e^- j_q^+ + j_e^+ j_q^- - j_e^\perp j_q^\perp$$

- ↪ electron “sees” (for  $p_e^z \rightarrow -\infty$ ) only  $j_q^+$

# Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates —  $\hat{z}$ -axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the  $j^+ = j^0 + j^z$  component of the quark current
- If up-quarks have positive orbital angular momentum in the  $\hat{x}$ -direction, then  $j^z$  is positive on the  $+\hat{y}$  side, and negative on the  $-\hat{y}$  side



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- ↪  $j^+$  is distorted not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to  $j^+$ ) “see” the quarks on the  $+\hat{y}$  side better than on the  $-\hat{y}$  side (for  $L_x > 0$ ).



# Transversely Distorted Distributions and $E(x, 0, -\Delta_{\perp}^2)$

- $q(x, \mathbf{b}_{\perp})$  in  $\perp$  polarized nucleon is distorted compared to longitudinally polarized nucleons !
- mean  $\perp$  displacement of flavor  $q$  ( $\perp$  flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2 \mathbf{b}_{\perp} q_X(x, \mathbf{b}_{\perp}) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

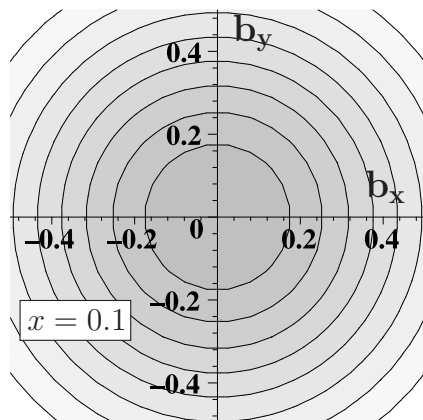
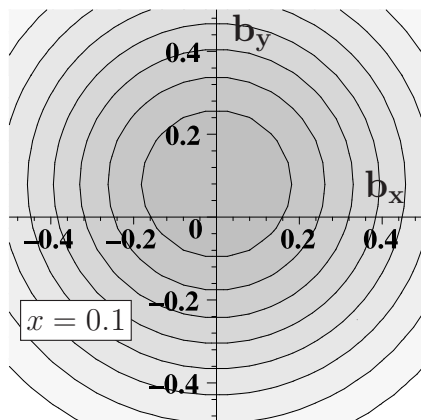
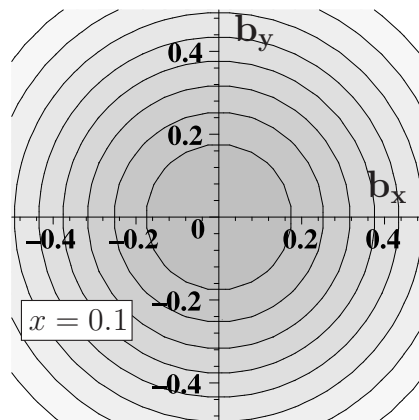
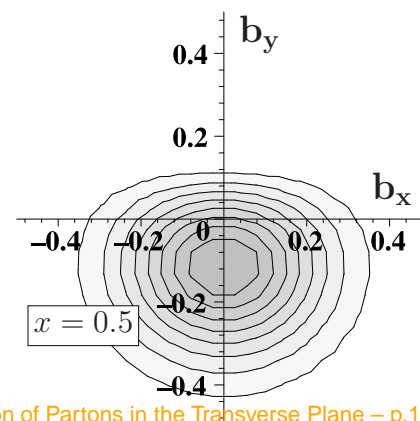
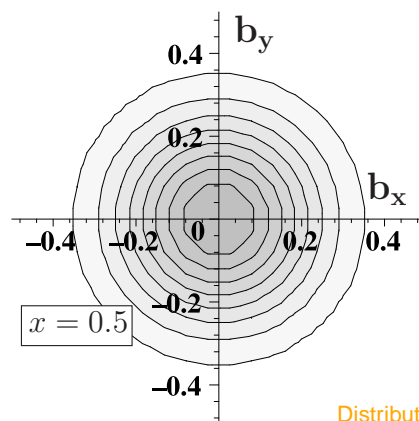
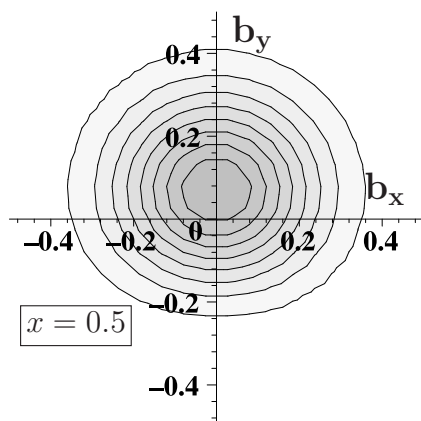
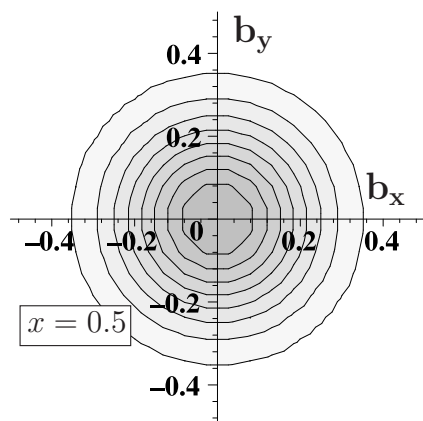
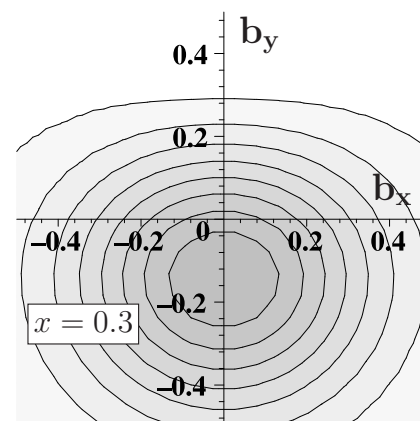
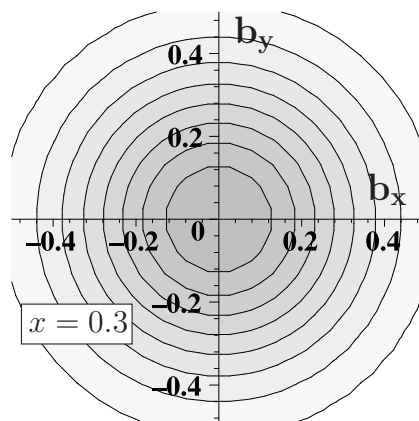
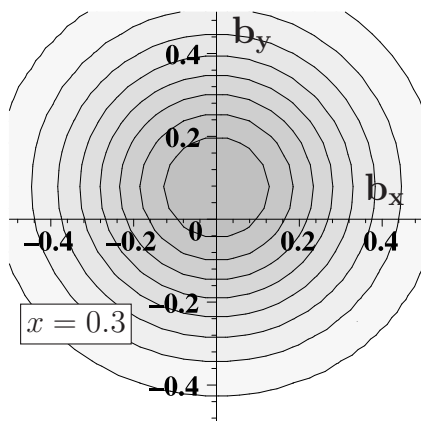
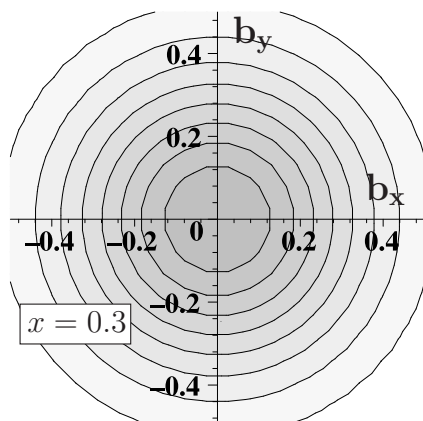
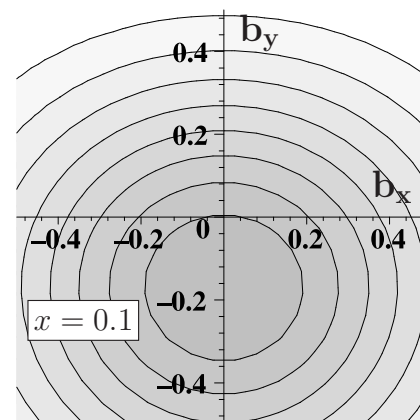
with  $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1 - 2) \Rightarrow d_y^q = \mathcal{O}(0.2 fm)$

- simple model: for simplicity, make ansatz where  $E_q \propto H_q$

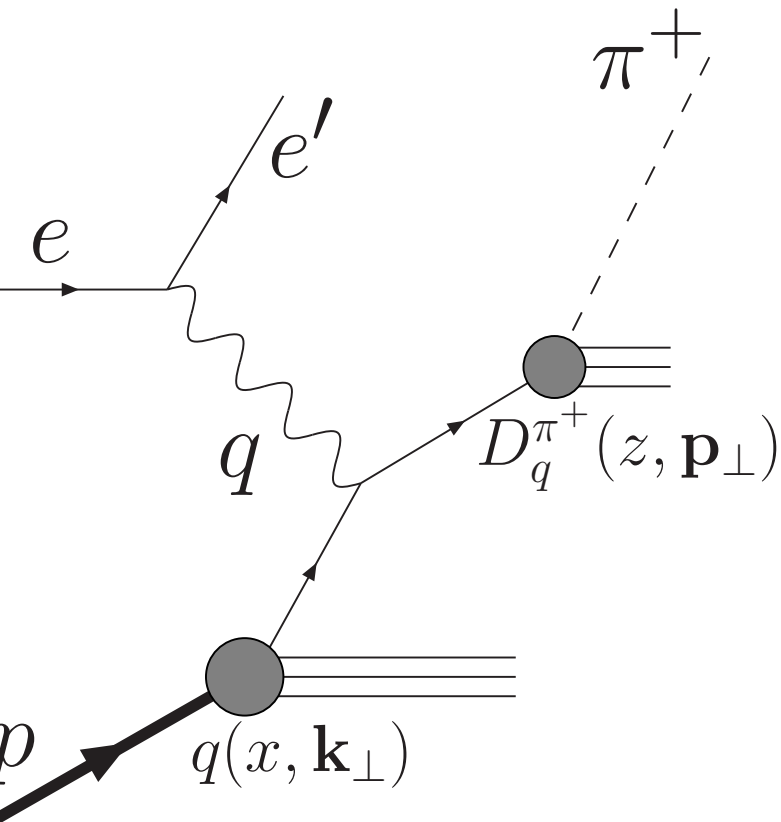
$$\begin{aligned} E_u(x, 0, -\Delta_{\perp}^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\Delta_{\perp}^2) \\ E_d(x, 0, -\Delta_{\perp}^2) &= \kappa_d^p H_d(x, 0, -\Delta_{\perp}^2) \end{aligned}$$

with  $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$        $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$ .

- Model too simple but illustrates that anticipated distortion is very significant since  $\kappa_u$  and  $\kappa_d$  known to be large!

$u(x, \mathbf{b}_\perp)$  $u_X(x, \mathbf{b}_\perp)$  $d(x, \mathbf{b}_\perp)$  $d_X(x, \mathbf{b}_\perp)$ 

# SSA ( $\gamma + p \uparrow \longrightarrow \pi^+ + X$ )



● use factorization (high energies) to express momentum distribution of outgoing  $\pi^+$  as **convolution** of

● momentum distribution of quarks in nucleon

↪ **unintegrated parton density**  $q(x, \mathbf{k}_\perp)$

● momentum distribution of  $\pi^+$  in jet created by leading quark  $q$

↪ **fragmentation function**  $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

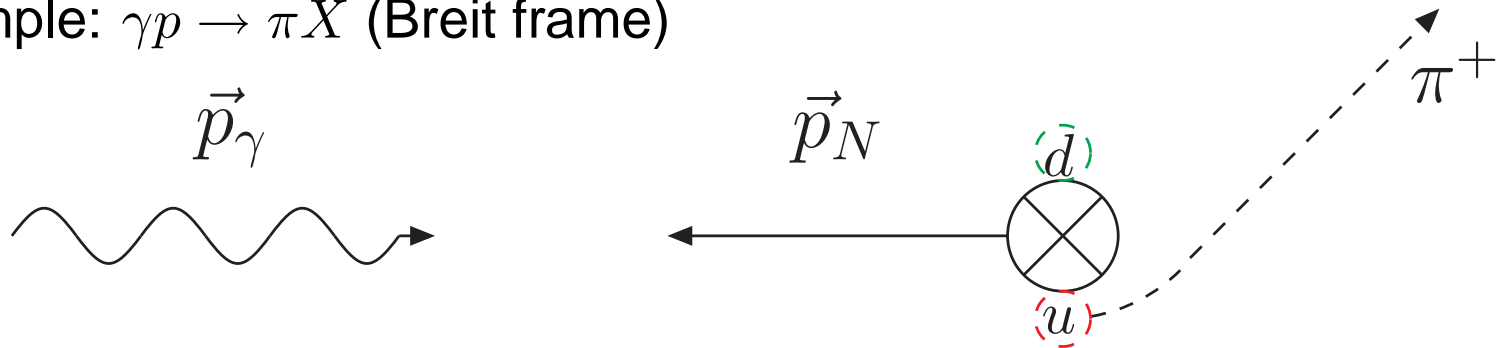
● average  $\perp$  momentum of pions obtained as sum of

● average  $\mathbf{k}_\perp$  of quarks in nucleon (Sivers effect)

● average  $\mathbf{p}_\perp$  of pions in quark-jet (Collins effect)

# GPD $\longleftrightarrow$ SSA (Sivers)

- example:  $\gamma p \rightarrow \pi X$  (Breit frame)



- $u, d$  distributions in  $\perp$  polarized proton have left-right asymmetry in  $\perp$  position space (T-even!); sign determined by  $\kappa_u$  &  $\kappa_d$
- attractive FSI deflects active quark towards the center of momentum
- $\hookrightarrow$  FSI translates position space distortion (before the quark is knocked out) in  $+\hat{y}$ -direction into momentum asymmetry that favors  $-\hat{y}$  direction
- naturally leads to correlation between sign of  $\kappa_q$  and sign of SSA:  

$$f_{1T}^{\perp q} \sim -\kappa_q$$
- signs of SSA confirmed at HERMES

# Chirally Odd GPDs

$$\int \frac{dx^-}{2\pi} e^{ixp^+ x^-} \left\langle p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \sigma^{+j} \gamma_5 q \left( \frac{x^-}{2} \right) \right| p \right\rangle = H_T \bar{u} \sigma^{+j} \gamma_5 u + \tilde{H}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha P_\beta}{M^2} u + E_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u$$

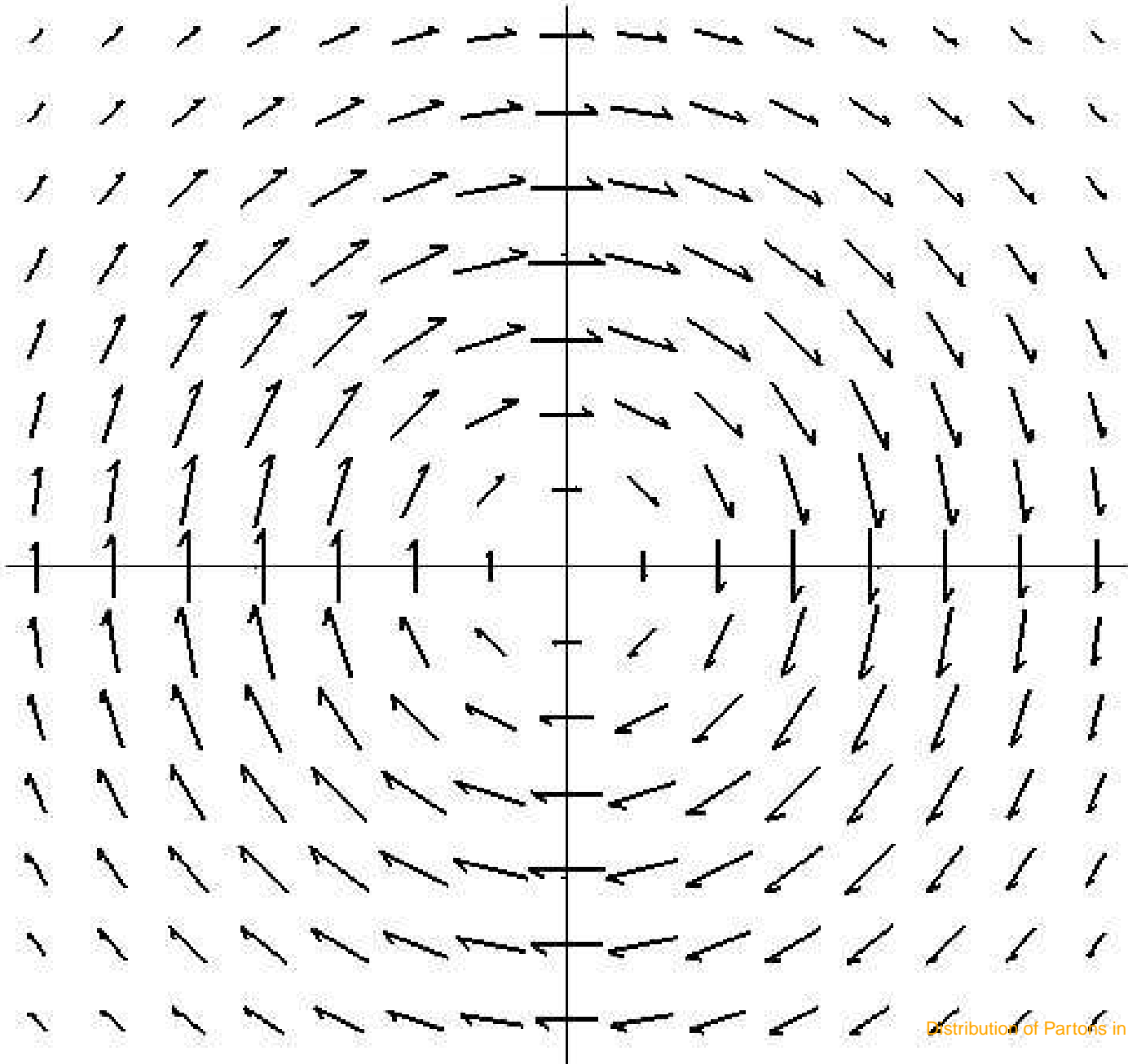
- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of  $\tilde{H}_T^q + 2E_T^q$  for  $\xi = 0$  describes distribution of transversity for unpolarized target in  $\perp$  plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \Delta_\perp} \left[ 2\tilde{H}_T^q(x, 0, -\Delta_\perp^2) + E_T^q(x, 0, -\Delta_\perp^2) \right]$$

- origin: correlation between quark spin (i.e.  $\perp$  component of transversity) and angular momentum

$$\int dx \left[ 2\tilde{H}_T^q(x, 0, 0) + E_T^q(x, 0, 0) \right] x = \langle J^i \rangle \text{ for quarks with spin } i$$

# Transversity Distribution in Unpolarized Target



# Transversity Distribution in Unpolarized Target

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry
  - ↪ e.g. quarks at negative  $b_x$  with spin in  $+\hat{y}$  get deflected (due to FSI) into  $+\hat{x}$  direction
  - ↪ (qualitative) connection between Boer-Mulders function  $h_1^\perp(x, \mathbf{k}_\perp)$  and the chirally odd GPD  $2\tilde{H}_T + E_T$  that is similar to (qualitative) connection between Sivers function  $f_{1T}^\perp(x, \mathbf{k}_\perp)$  and the GPD  $E$ .
  - ↪ qualitative predictions for  $h_1^\perp(x, \mathbf{k}_\perp)$ 
    - sign of  $h_1^\perp$  opposite sign of  $2\tilde{H}_T + E_T$
    - “ $\frac{h_1^\perp}{2\tilde{H}_T + E_T} \approx \frac{f_{1T}^\perp}{E}$ ”
- use measurement of  $h_1^\perp$  to learn about spin-orbit correlation in nucleon wave function
- use LGT calcs. of  $2\tilde{H}_T + E_T$  to make qualitative prediction for  $h_1^\perp$

# Summary

- GPDs provide decomposition of form factors w.r.t. the momentum of the active quark

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlator, but  $\Delta \equiv p' - p \neq 0$ .
- $t$ -dependence of GPDs at  $\xi=0$  (purely  $\perp$  momentum transfer)  $\Rightarrow$  Fourier transform of **impact parameter dependent PDFs**  $q(x, \mathbf{b}_\perp)$
- $\hookrightarrow$  knowledge of GPDs for  $\xi = 0$  provides novel information about nonperturbative parton structure of nucleons: **distribution of partons in  $\perp$  plane**

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{i\mathbf{b}_\perp \cdot \Delta_\perp}$$
$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\Delta_\perp^2) e^{i\mathbf{b}_\perp \cdot \Delta_\perp}$$

- $q(x, \mathbf{b}_\perp)$  has probabilistic interpretation, e.g.  $q(x, \mathbf{b}_\perp) > 0$  for  $x > 0$



# Summary

- $\frac{\Delta_{\perp}}{2M} E(x, 0, -\Delta_{\perp}^2)$  describes how the momentum distribution of unpolarized partons in the  $\perp$  plane gets transversely distorted when is nucleon polarized in  $\perp$  direction.
- (attractive) final state interaction in semi-inclusive DIS converts  $\perp$  position space asymmetry into  $\perp$  momentum space asymmetry
- simple physical explanation for observed Sivers effect in  $\gamma^* p \rightarrow \pi X$
- $2\tilde{H}_T + E_T$  measures correlation between  $\perp$  spin and  $\perp$  angular momentum
- physical explanation for Boer-Mulders mechanism; suggest new relation between  $h_1^{\perp}$  and the GPDs  $2\tilde{H}_T + E_T$
- published in: M.B., PRD **62**, 71503 (2000), Int. J. Mod. Phys. **A18**, 173 (2003); see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **69**, 057501 (2004); NPA **735**, 185 (2004); PRD **66**, 114005 (2002).